Influence of vortical disturbances on flame-acoustics interactions

Raphael Assier[1], Xuesong Wu[2]
Imperial College, Department of Mathematics, SW7 2RH London, UK,
[1]*Email: r.assier@imperial.ac.uk,* [2]*Email: x.wu@imperial.ac.uk*

Introduction

The problem of premixed combustion in a duct is investigated using an asymptotic formulation, which is derived from first principles and based on low Mach number and high activation energy assumptions [6]. Contrary to other flame models, such as the Michelson-Sivashinsky equation or the G-equation, the present approach is complete in the sense that it takes into account the interactions between the flame and the spontaneous acoustic field, as well as the interactions between the hydrodynamic field and the flame. The focus is on the fundamental mechanisms of combustion instability. To this end, a linear stability analysis of some steady curved flames is carried out. These steady flames were known to be stable when the spontaneous acoustic perturbations are ignored, but we have shown that they are actually unstable when the latter effect is included. In order to corroborate this result, a coupled weakly non-linear numerical simulation is implemented. The linear instability result is confirmed by the numerical study. In the present work, motivated by [5], we also study the effect of vortical disturbances in the oncoming fresh mixture. These vortical disturbances cause wrinkling of the flame, and resonance between flame oscillations and acoustic fluctuations.

Problem formulation and asymptotic analysis

We consider the problem of pre-mixed combustion in a duct. We consider the combustion as a one-step irreversible chemical reaction, where the fuel is the deficient reactant (lean combustion). The mixture is assumed to obey the state equation for a perfect gas and the fluid is assumed to be Newtonian. Following [6], we consider five governing equations, namely the equations of conservation of mass, momentum and energy, as well as the state equation and the transport equation governing the diffusion of chemical species. The problem is then non-dimensionalised. The space variables are scaled by $h^* = h/2$, where h is the width of the duct. The density and the temperature are scaled by $\rho_{-\infty}$ and $\theta_{-\infty}$, the density and temperature in the fresh mixture. The velocities are scaled by U_L, the laminar speed of flame propagation. The geometry of the problem is described in Figure 1.

This process of non-dimensionalisation leads to a system of five governing equations, in which δ represents the flame thickness, G represents the gravity term, β the activation energy, M the Mach number and q the mean heat release. Following [2], the flame front is defined as a flame discontinuity in the hydrodynamic field by $x = f(y, z, t)$ (see Figure 1). In order to track this flame front, we perform a change of variable to express the problem in the flame frame of reference. We pass from the variables (x, y, z, t) to the variables (ξ, η, ζ, τ), where $\xi = x - f(y, z, t)$, $\eta = y$, $\zeta = z$ and $\tau = t$. By expressing the velocity field as $\vec{u} = u\vec{e_\xi} + \vec{v}$ and considering a reduced gradient $\widetilde{\nabla} = (\partial/\partial\eta \quad \partial/\partial\zeta)^T$, we obtain a new system of equations in the flame frame of reference. The new geometry of the problem is described in Figure 2.

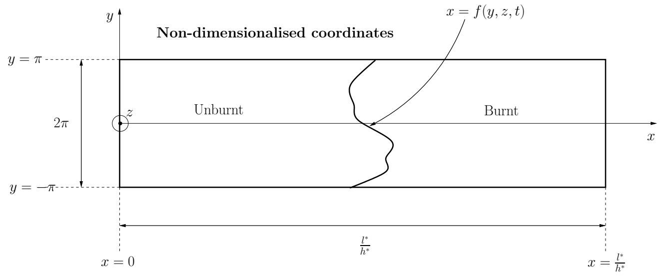

Figure 1: Non-dimensionalised geometry of the problem

In order to simplify the problem, we make three main asymptotic assumptions: a large activation energy ($\beta \gg 1$), a small flame thickness ($\delta \ll 1$) and a low Mach number ($M \ll 1$). The inequality $\delta/\beta \ll \delta \ll 1 \ll 1/M$ results in four asymptotic zones described in Figure 2: the reaction zone where most of the chemistry occurs, the pre-heated zone where thermal diffusion is important, the hydrodynamic zone and the acoustic zone.

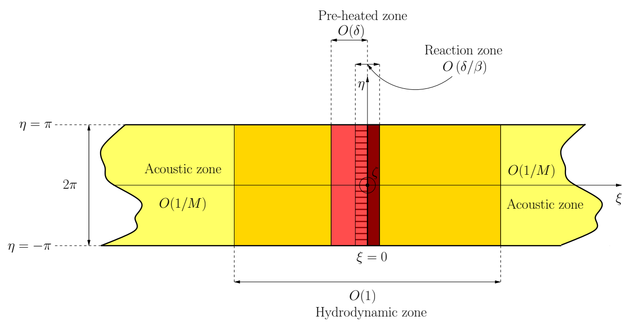

Figure 2: Flame frame of reference geometry and different asymptotic zones

A careful asymptotic analysis leads to the definition of the acoustic velocity and acoustic pressure $u_a\left(\tilde{\xi}, \tau\right)$ and $p_a\left(\tilde{\xi}, \tau\right)$, where $\tilde{\xi}$ is the stretched longitudinal variable defined by $\tilde{\xi} = M\xi$. This acoustic field is described by the following system of equations:

$$\left\{ \begin{array}{rcl} \frac{\partial p_a}{\partial \tau} + \frac{\partial u_a}{\partial \tilde{\xi}} & = & 0 \\ R\frac{\partial u_a}{\partial \tau} + \frac{\partial p_a}{\partial \tilde{\xi}} & = & 0 \end{array} \right. , \quad (1)$$

where $\rho = R + O(\delta)$. It is also possible to derive the following jump conditions across the hydrodynamic zone:

$$\left\{ \begin{array}{rcl} [[p_a]]_-^+ & = & 0 \\ [[u_a]]_-^+ & = & \mathcal{J}_a(\tau) = \frac{q}{2}\overline{\left(\tilde{\nabla} F\right)^2} \end{array} \right. , \quad (2)$$

where the overbar represents a space average and $[[\quad]]_-^+$ represents the jump across the hydrodynamic zone and $f = F + O(\delta)$. By introducing new hydrodynamic variables $U, \vec{V}$ and P defined by $(u, \vec{v}, p) = \left(U, \vec{V}, P\right) + O(\delta)$ into the governing equations and partially linearising the result, one obtains a system of equations describing the hydrodynamic field:

$$\left\{ \begin{array}{rcl} \frac{\partial U}{\partial \xi} + \tilde{\nabla} \cdot \vec{V} & = & 0 \\ R\frac{\partial U}{\partial \tau} + \frac{\partial U}{\partial \xi} & = & -\frac{\partial P}{\partial \xi} \\ R\frac{\partial \vec{V}}{\partial \tau} + \frac{\partial \vec{V}}{\partial \xi} & = & -\tilde{\nabla} P \end{array} \right. \quad (3)$$

and the jump conditions across the flame front:

$$\left\{ \begin{array}{rcl} [U]_-^+ & = & 0 \\ \left[\vec{V}\right]_-^+ & = & -q\tilde{\nabla} F \\ [P]_-^+ & = & -\left(\mathcal{L}_a(\tau) + \frac{qG}{1+q}\right) F \end{array} \right. , \quad (4)$$

where $[\quad]_-^+$ represents the jump across the flame and $\mathcal{L}_a(\tau)$ is a term of acoustic acceleration defined by $\mathcal{L}_a(\tau) = \left[\left[\frac{\partial p_a}{\partial \tilde{\xi}}\right]\right]_-^+$. Finally, the flame front is described by the so-called flame equation:

$$\frac{\partial F}{\partial \tau} = U\left(0^-, \eta, \zeta\right) - \frac{1}{2}\left(\tilde{\nabla} F\right)^2 + \delta\nu\tilde{\nabla}^2 F, \quad (5)$$

where ν represents the Markstein length of the problem.

Linear stability analysis of curved steady solutions

Considering the simplified 2D problem (i.e the variables of the problem are now (ξ, η, τ)) and using Fourier analysis, it is possible to compute some curved steady solutions of the flame equation (5). It can be shown (see [1]) that when neglecting gravity (i.e. $G = 0$), the steady solutions obtained correspond to the steady N-pole coalescent solutions of the Michelson-Sivashinsky equation. These steady solutions have been proved to be stable in [3] and [4] where acoustic fluctuations are artificially ignored. An important parameter arising from this study is the parameter γ defined by $\gamma = q/(\delta\nu)$. Performing a linear stability analysis of these curved steady states with our complete model (i.e. including acoustic fluctuations) results in a non-linear eigenvalue problem depending on the mean position σ of the flame in the duct. In order to simplify the problem, we linearise around the first acoustic mode $\omega_1(\sigma)$ of the duct. As seen in Figure 3 and demonstrated in [1], solving the resulting linear eigenvalue problem for each value of σ shows that the curved steady solutions are actually linearly unstable when the spontaneous acoustic fluctuations are taken into account.

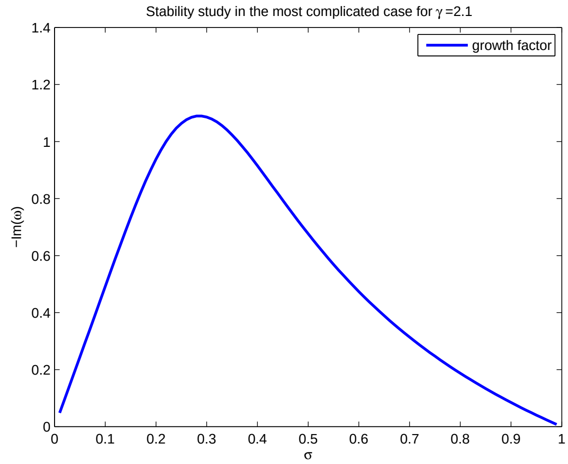

Figure 3: Result of the linear stability analysis for $\gamma = 2.1$

Spectral flame equation and numerical resolution

Starting from the system $\{(3),(4),(5)\}$, and using Fourier analysis, it is possible to summarise everything in one single equation. This equation, the spectral flame equation, lies in the spectral k-space arising when taking the Fourier transform in the η-direction.

$$\begin{aligned} & A\frac{\partial^2 \hat{F}}{\partial \tau^2} + B(k)\frac{\partial \hat{F}}{\partial \tau} + C(k,\tau)\hat{F} \\ & = -|k|\left(ik'\hat{F}(k')\right) \star \left(ik'\hat{F}(k')\right)(k) \\ & - A\left(ik'\hat{F}(k')\right) \star \left(ik'\frac{\partial \hat{F}}{\partial \tau}(k')\right)(k) + \mathcal{N}_0(k,\tau), \end{aligned} \quad (6)$$

where A, $B(k)$ and $C(k,\tau)$ are known coefficients depending on k and the only time dependency in $C(k,\tau)$ is directly related to the quantity $\mathcal{L}_a(\tau)$. The hat symbol $\widehat{\ }$ represents the Fourier transform in the η direction and the star symbol $\star$ represents the convolution in the k-space. Finally, $\mathcal{N}_0(k,\tau)$ represents a forcing term characterising the presence of vortical disturbances at the inlet.

In order to corroborate the results of the linear stability analysis, a numerical scheme has been developed. The first thing to do is to solve the acoustic system $\{(1),(2)\}$ for a given function of time $\mathcal{J}_a(\tau)$. This is done with a semi-analytic method based on the method of characteristics. The second important task is to solve the spectral flame equation (6) for a given function of time $\mathcal{L}_a(\tau)$. This is done by considering it as an initial value problem, where the initial condition is given by

$$\begin{cases} \hat{F}(k,0) & = & \widehat{F_0}(k) \\ \frac{\partial \hat{F}}{\partial \tau}(k,0) & = & 0 \end{cases} , \tag{7}$$

where $\widehat{F_0}(k)$ is the Fourier transform of the slightly perturbed steady state of interest. This initial value problem is then solved numerically, evaluating the convolution using the Fast Fourier Transform algorithm. Hence at this stage, if we know $\mathcal{J}_a(\tau)$, we can solve the acoustic system, and if we know $\mathcal{L}_a(\tau)$, we can solve the spectral flame equation. The last step consists of coupling these two methods so that at each time step, the acoustic and the spectral scheme communicate in order to evaluate accurately the values of $\mathcal{J}_a(\tau)$ and $\mathcal{L}_a(\tau)$. The Figures 4 to 6 show the numerical results obtained when no vortical disturbances are added to the system (i.e. $\mathcal{N}_0(k,\tau) \equiv 0$), $\gamma = 2.1$ and $\sigma = 0.5$. Figure 4 shows the evolution of $\mathcal{J}_a(\tau)$, Figure 5 shows the evolution of the acoustic pressure at the closed end of the tube, exhibiting clearly the exponential growth corresponding to the linear instability. In Figure 6, we present the "time" power spectrum of the acoustic pressure, showing the the first acoustic mode is indeed dominant. This justifies the linearisation assumption used in the linear analysis.

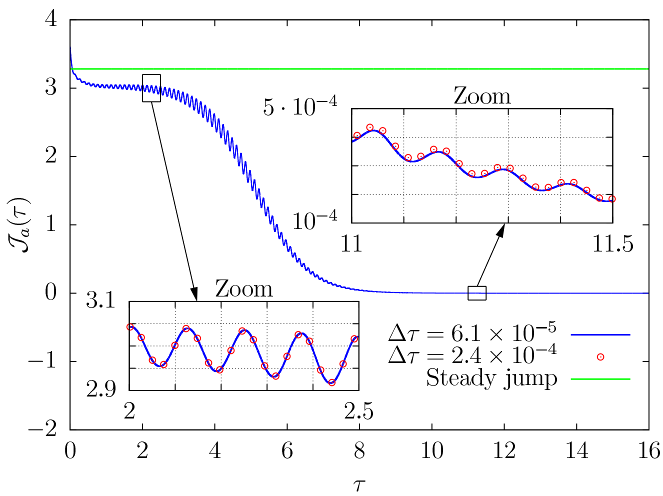

Figure 4: Evolution of the acoustic jump $\mathcal{J}_a(\tau)$

The evolution of the flame shape can be described in three steps: at the beginning, the flame remains close to the steady state, and starts oscillating up and down. Meanwhile the acoustic pressure starts to amplify exponentially. The flame starts to flatten while still oscillating due to the stabilizing effect of the pressure. Finally, when the acoustic pressure saturates, the flame becomes completely flat and remains so.

Finally, Figure 7 shows the agreement between the growth rate measured from the numerical results (obtained for two different initial way of disturbing the steady state) and the prediction of the linear stability analysis.

A very interesting interpretation of these numerical results is that, apart from validating the results of the

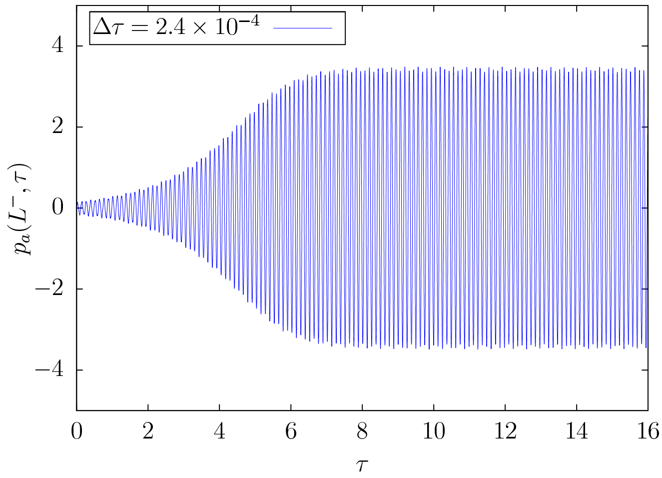

Figure 5: Evolution of the acoustic pressure at the closed end of the duct

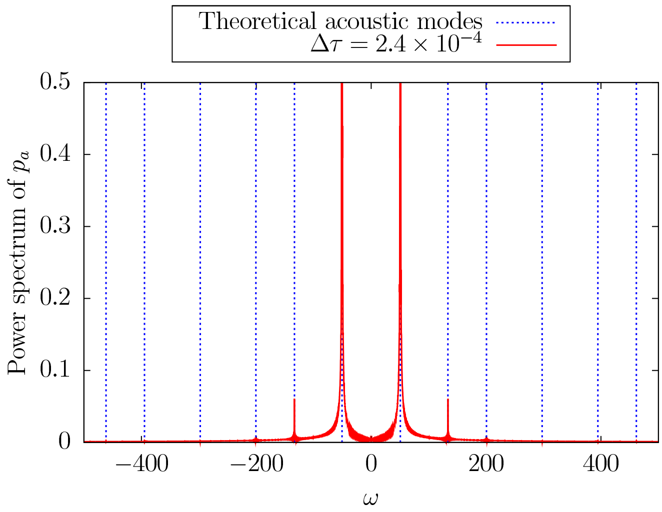

Figure 6: Power spectrum of the acoustic pressure

linear stability analysis, they show that a flat flame, which is intrinsically unstable in a silent environment due to Darrieus-Landau instability, can survive and remain flat in a noisy environment created by its spontaneous sound.

Influence of vortical disturbances: a secondary instability

The term $\mathcal{N}_0(k,\tau)$ representing vortical disturbances naturally takes the form

$$\mathcal{N}_0(k,\tau) = \varepsilon e^{-(|k|-k_0)^2} \{(-iR_-\omega + |k|)\, e^{-i\omega\tau} + c.c.\} \tag{8}$$

when deriving the spectral flame equation. k_0 represents the wave number of the disturbance (here $k_0 = 4$) and ω its frequency. As seen in Figure 8, when choosing $\omega = \omega_1/2$, the system exhibits a resonance phenomenon (it does not happen for any other frequencies). The plots of Figure 8 represent the envelope of the acoustic pressure at the closed end of the duct for different strength of vortical disturbances (i.e. different values of ε).

Here, it is again possible to describe the evolution of the flame shape in a few steps. First it remains close

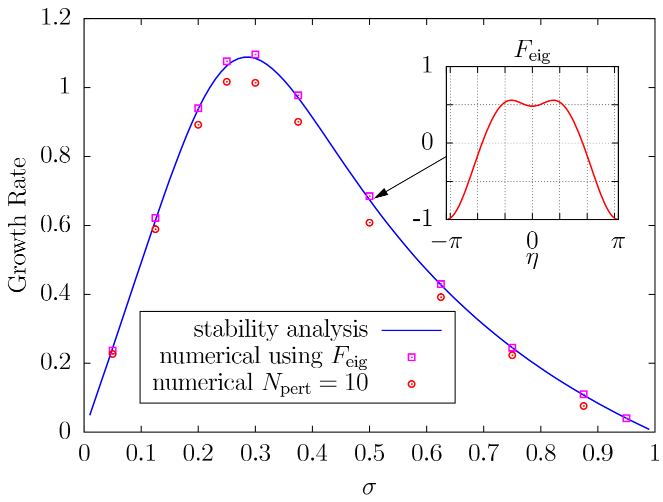

Figure 7: Comparison between numerical measurement and linear stability analysis prediction

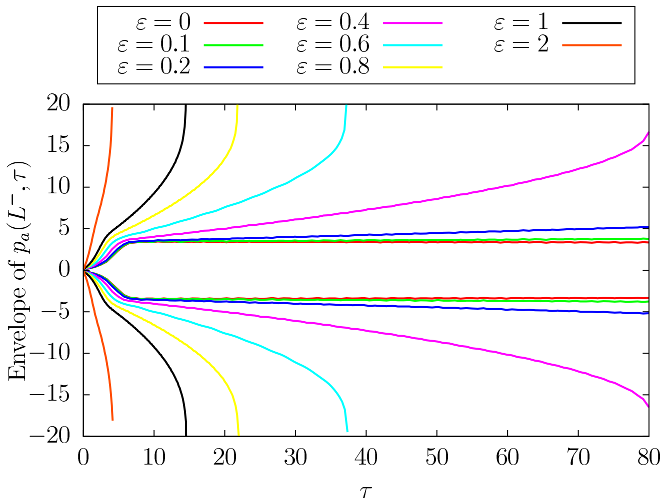

Figure 8: Envelope of the acoustic pressure for different strength of vortical disturbances

to the steady state and start oscillating up and down while being slightly wrinkled by the vortical disturbance. Then, even though the slight wrinkling persists, the overall shape of the flame tends to flatten, corresponding to the first exponential growth of the acoustic pressure. Following this, the flame remains wrinkled with a wave number k_0 and oscillate at the frequency ω around the flat state. After a while, these oscillations start to grow exponentially. This is due to the fact that the acoustic acceleration now induces a parametric resonance. A similar behaviour occurs with an initially flat flame as was shown in [5]. Hence by slightly disturbing the hydrodynamic field, we have triggered a secondary instability.

Conclusion

A new generation of flame model has been investigated analytically and implemented numerically, emphasising the fact that it is fundamental to consider acoustic fluctuation when modelling combustion. We have shown that a flat flame could be stable under the influence of its spontaneous sound. Finally, by adding vortical disturbances, we exhibited a secondary type of combustion instability based on a resonance between flame oscillations and acoustic fluctuations.

References

[1] R. C. Assier and X. Wu: Acoustic coupling in combustion instability.
In Proceedings of the 19th International Congress on Sound and Vibration, 1–8, 2012.

[2] M. Matalon and B. J. Matkowsky: Flames as gasdynamic discontinuities.
Journal of Fluid Mechanics, 124:239–259, 1982.

[3] D. Vaynblat and M. Matalon: Stability of pole solutions for planar propagating flames: I. exact eigenvalues and eigenfunctions.
SIAM J. Appl. Math., 60(2):679–702,2000.

[4] D. Vaynblat and M. Matalon: Stability of pole solutions for planar propagating flames: II. Properties of eigenvalues/eigenfunctions and implications to stability.
SIAM J. Appl. Math., 60(2):703–728, 2000.

[5] X. Wu and C. K. Law: Flame-acoustic resonance initiated by vortical disturbances.
Journal of Fluid Mechanics, 634:321–357, 2009.

[6] X. Wu, M. Wang, P. Moin, and N. Peters: Combustion instability due to the nonlinear interaction between sound and flame.
Journal of Fluid Mechanics, 497:23–53, 2003.