Electromagnetic Wave Diffraction of Perfect Electric Conducting Wedges with Arbitrary Linear Polarization

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Abstract

This paper focuses on finding the electromagnetic (EM) field and the time-averaged Poynting vector produced after a time harmonic EM plane wave of an arbitrary fixed (linear) polarization is incident on an infinite perfect electric conducting (PEC) wedge. The aim is to find out how the polarization of this incident EM wave impacts the solution to diffraction by perfectly conducting wedges.

We use the z invariance of the scatterer and the PEC boundary conditions to rewrite the EM field governed by Maxwell's equations in terms of two uncoupled scalar potentials or Debye potentials. These potentials will be functions of an arbitrary polarization angle and respectively solve the acoustic (or scalar) wedge problem with Dirichlet or Neumann boundary conditions.

Keywords: Electromagnetic Wave Diffraction, PEC Wedge, Arbitrary Linear Polarization, Debye Potential, Sommerfeld-Malyuzhinets Technique

1 Introduction

The focus of this paper is the diffraction of a time harmonic EM plane wave of any polarization by a PEC infinite wedge. To solve this, we follow techniques in [5], [4] and [9] to rewrite the EM field for Maxwell's equations in terms of two uncoupled scalar potentials called the Debye potentials. These potentials will both solve the scalar infinite wedge problem with perfect boundary conditions, i.e. Dirichlet or Neumann boundary conditions. We find these scalar solutions by the Sommerfeld-Malyuzhinet technique outlined in [1]. We check the scalar wedge solutions by comparing with results and plots obtained in [1] and [3]. The most comparable paper is [6] which studies an inhomogeneous (or evanescent) EM plane wave of arbitrary polarization diffracted by a PEC wedge at skew incidence expanding on a simpler problem in [7]. Introduced in 1909 by P. Debye in [2], Debye potentials have mostly been used for problems involving conical or spherical domains, for example [10] and [8]. These same methods can also be applied to wedge problems.

2 Formulation

Let the region exterior to the wedge be $-\pi < -\theta_w < \theta < \theta_w < \pi$, where $\theta = \pm\theta_w$ are the wedge faces. The incident wave is a time-harmonic EM plane wave with time factor $e^{-i\omega t}$, wavenumber k, amplitude $\mathcal{A}$, polarization angle α and incident angle $\theta = \theta_I$. The governing equations are Maxwell's equations. We assume that the domain is linear, isotropic, homogeneous and source free so that the electric permittivity ϵ and the magnetic permeability μ can be assumed to be constant scalars. The electric and magnetic phasors, $\underline{E}$ and $\underline{H}$, are defined from the electric and magnetic intensity, $\underline{\hat{E}}$ and $\underline{\hat{H}}$, by,

$$\underline{\hat{E}} = \mathrm{Re}\left\{\frac{\underline{E}e^{-i\omega t}}{\sqrt{\epsilon}}\right\}, \quad \underline{\hat{H}} = \mathrm{Re}\left\{\frac{\underline{H}e^{-i\omega t}}{\sqrt{\mu}}\right\}. \quad (1)$$

Noting that the wave speed is $c = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}$, Maxwell's equations can be rewritten in terms of the two phasors,

$$\begin{aligned} \nabla \times \underline{H} + ik\underline{E} = \underline{0}, \quad \nabla \cdot \underline{E} = 0, \\ \nabla \times \underline{E} - ik\underline{H} = \underline{0}, \quad \nabla \cdot \underline{H} = 0. \end{aligned} \quad (2)$$

The PEC boundary conditions are,

$$\underline{E} \times \underline{n}|_{\theta=\pm\theta_w} = \underline{0}, \quad (3)$$

$$\underline{H} \cdot \underline{n}|_{\theta=\pm\theta_w} = 0, \quad (4)$$

where $\underline{n}$ is the unit normal to the wedge faces,

$$\underline{n}|_{\theta=\pm\theta_w} = \pm\underline{e_\theta}. \quad (5)$$

These boundary conditions imply that the electric field has no tangential component on the wedge faces and that both the magnetic field and the Poynting vector,

$$\underline{\hat{S}} = \underline{\hat{E}} \times \underline{\hat{H}}, \quad (6)$$

have no normal component on the wedge faces. We also define the time-averaged Poynting vector in terms of the phasors as,

$$\underline{S} = \frac{1}{2}c\Re\{\underline{E} \times \underline{H}^*\}. \quad (7)$$

3 The Electromagnetic Field

The EM field satisfying (2)-(4) can be written in terms of an electric and a magnetic vector potential which are both independent of z and fixed in the z direction requiring us to construct two scalar potentials. These two scalar potentials are proportional to the total field solutions to the acoustic wedge problem with Dirichlet and Neumann boundary conditions, denoted $\Phi^{(D)}$ and $\Phi^{(N)}$ respectively. This EM field is,

$$\underline{E} = \begin{pmatrix} -\frac{\sin(\alpha)}{ikr}\frac{\partial}{\partial\theta}\Phi^{(N)} \\ \frac{\sin(\alpha)}{ik}\frac{\partial}{\partial r}\Phi^{(N)} \\ \cos(\alpha)\Phi^{(D)} \end{pmatrix}, \quad (8)$$

$$\underline{H} = \begin{pmatrix} \frac{\cos(\alpha)}{ikr}\frac{\partial}{\partial\theta}\Phi^{(D)} \\ -\frac{\cos(\alpha)}{ik}\frac{\partial}{\partial r}\Phi^{(D)} \\ \sin(\alpha)\Phi^{(N)} \end{pmatrix}. \quad (9)$$

where α is the polarization angle and denotes the angle that the incident electric field makes with the z-axis. $\Phi^{(D)}$ and $\Phi^{(N)}$, obtained from [1], in integral form are,

$$\Phi^{(D)} = \frac{\mathcal{A}}{2\pi i}\int_{\gamma_+ + \gamma_-} \frac{\delta\cos(\delta\theta_I)e^{-ikr\cos(\hat{z})}}{\sin(\delta(\theta+\hat{z})) - \sin(\delta\theta_I)}d\hat{z} \quad (10)$$

$$\Phi^{(N)} = \frac{\mathcal{A}}{2\pi i}\int_{\gamma_+ + \gamma_-} \frac{\delta\cos(\delta(\theta+\hat{z}))e^{-ikr\cos(\hat{z})}}{\sin(\delta(\theta+\hat{z})) - \sin(\delta\theta_I)}d\hat{z} \quad (11)$$

where $\delta = \frac{\pi}{2\theta_w}$ and γ_+, γ_- are the usual Sommerfeld contours.

4 Conclusion

The EM field can be approximated as $kr \to \infty$ for a high frequency or far-field approximation. This is used to determine $\underline{E} \cdot \underline{H}$ and $\underline{S}$. We find that the problem is $\underline{E}$-polarized if $\alpha = 0$ or π and is $\underline{H}$-polarized if $\alpha = \pm\frac{\pi}{2}$. In both cases the total EM field is orthogonal and the Poynting vector is confined to the x-y plane. If α is not equal to a multiple of $\frac{\pi}{2}$ then the total EM field is not orthogonal and the Poynting vector is not confined to the x-y plane.

References

[1] V. M. Babich, M. A. Lyalinov, and V. E. Grikurov. *Diffraction Theory: The Sommerfeld-Malyuzhinets Technique.* Alpha Science, 2007.

[2] P. Debye. Der Lichtdruck auf Kugeln von beliebigem material. *Annals of Physics*, 335(11):57–136, 1909.

[3] F. Hacivelioglu, L. Sevgi, and P. Y. Ufimtsev. Electromagnetic Wave Scattering from a Wedge with Perfectly Reflecting Boundaries : Analysis of Asymptotic Techniques. *IEEE Antennas and Propagation Magazine*, 53(3):232–253, 2011.

[4] G. L. James. *Geometrical Theory of Diffraction for Electromagnetic Waves.* Peter Peregrinus, The Institution of Engineering and Technology, London, third edition, 1986.

[5] D. S. Jones. *The Theory of Electromagnetism.* Pergamon Press, London, 1964.

[6] R. G. Kouyoumjian, T. Celandroni, G. Manara, and P. Nepa. Inhomogeneous electromagnetic plane wave diffraction by a perfectly electric conducting wedge at oblique incidence. *Radio Science*, 42(6):295–304, 2007.

[7] R. G. Kouyoumjian, G. Manara, P. Nepa, and B. J. E. Taute. The Diffraction of an Inhomogeneous Plane Wave by a Wedge. *Radio Science*, 31(6):1387–1397, 1996.

[8] M. A. Lyalinov. Electromagnetic scattering by a plane angular sector : I . Diffraction coefficients of the spherical wave from the vertex. *Wave Motion*, 55:10–34, 2015.

[9] M. A. Lyalinov and N. Y. Zhu. *Scattering of Waves by Wedges and Cones with Impedance Boundary Conditions.* Scitech, Edison, New Jersey, 2013.

[10] V. P. Smyshlyaev. The High Frequency diffraction of Electromagnetic Waves by Cones of Arbitrary Cross Sections. *SIAM Journal of Applied Mathematics*, 53(3):670–688, 1993.